Atom Interferometry 101

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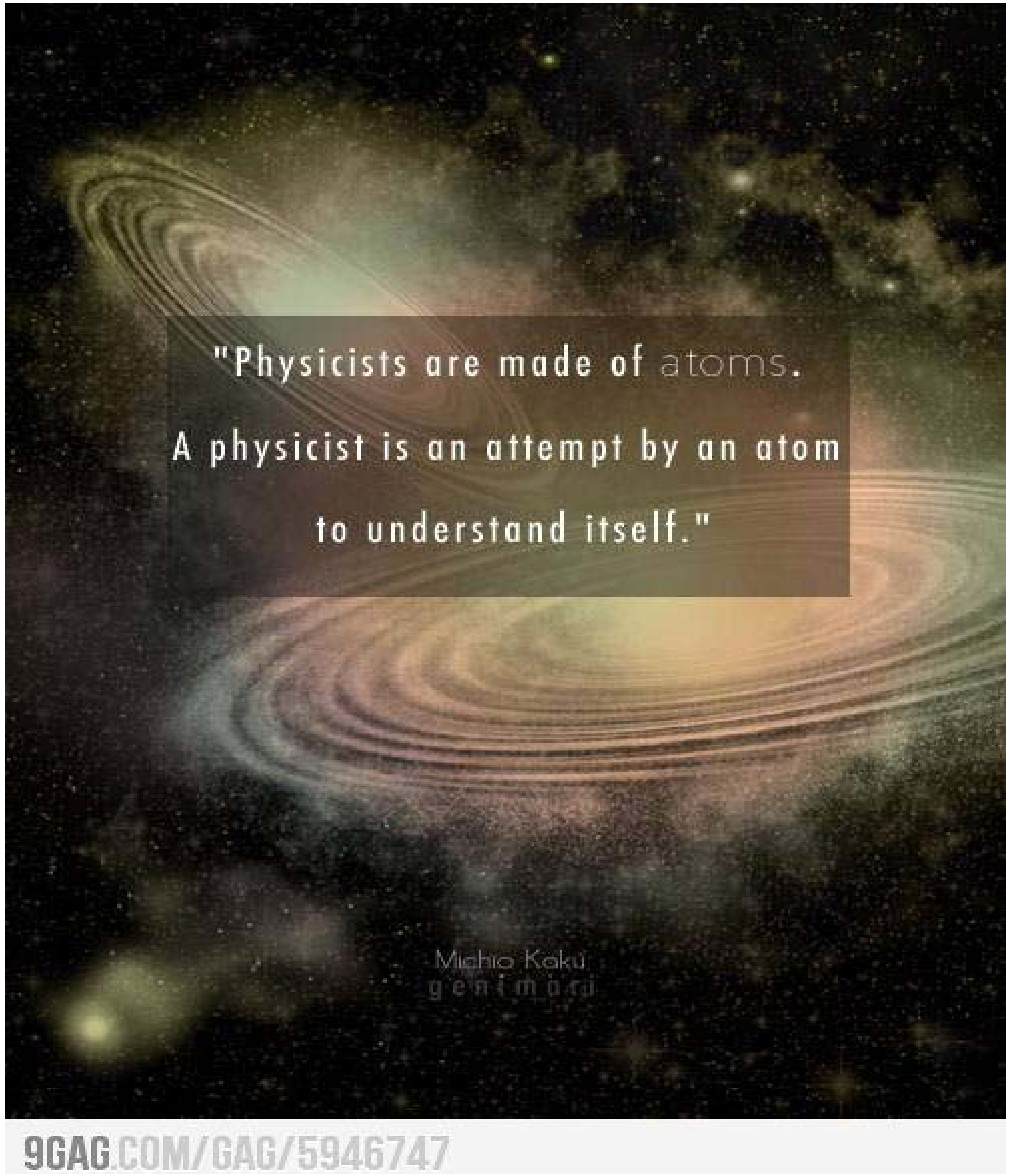
Atomic physics (for the lay person)



History

- deBroglie proposal 1924

The Nobel Prize in Physics 1929 was awarded to Louis de Broglie "for his discovery of the wave nature of electrons".

- Electron diffraction 1930

The Nobel Prize in Physics 1937 was awarded jointly to Clinton Joseph Davisson and George Paget Thomson "for their experimental discovery of the diffraction of electrons by crystals"

- Electron interferometry 1950s
- Neutron interferometry 1960s

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•

- Atom interferometers 1990s

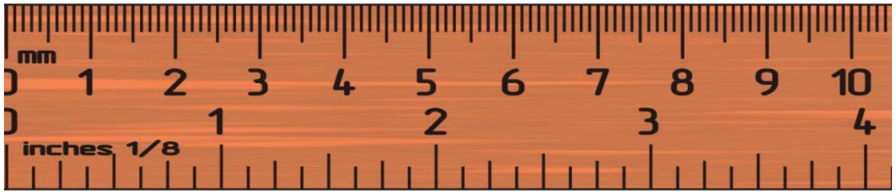
Internal structure!!!

More history

- deBroglie proposal 1924

$$\lambda = \frac{h}{mv}$$

The “ruler” for precision measurements



- Electron diffraction 1930
- Electron interferometry 1950s
- Neutron interferometry 1960s
- Atom interferometers 1990s

$$m_e = 9.10938188 \times 10^{-31} kg$$

X 1000

$$m_n = 1.6749 \times 10^{-27} kg$$

X 100

$$m_{Cs} = 2.2062 \times 10^{-25} kg$$

Why??

PHYSICAL REVIEW A VOLUME 48, NUMBER 4 OCTOBER 1993

Quantum-noise limits to matter-wave interferometry

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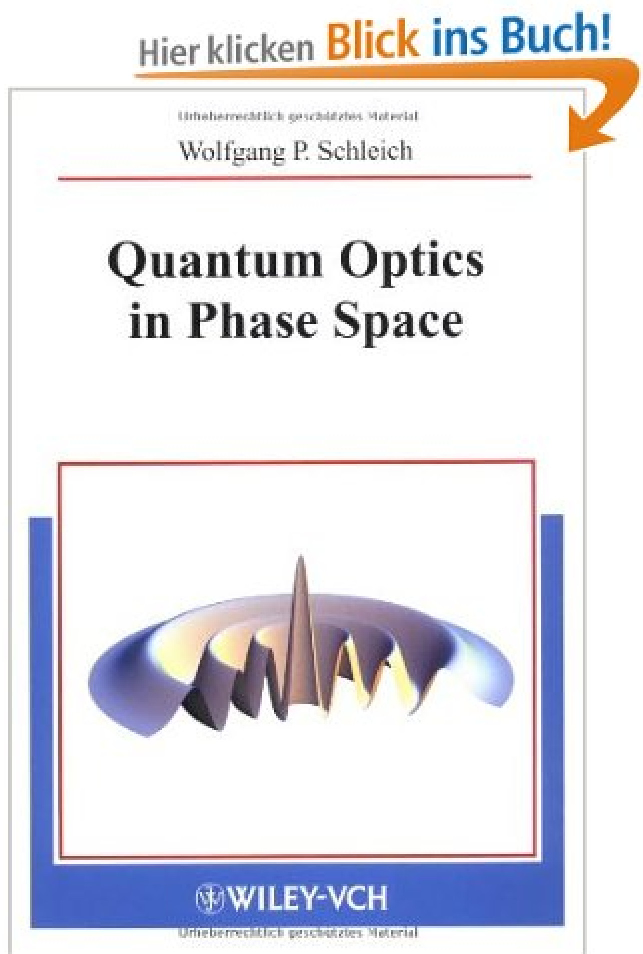
(Received 14 August 1992)

TABLE I. Compared and contrasted are different properties of matter-wave and optical gyroscopes in terms of their sensitivity to phase differences—or equivalently—rotation rates. We see that the high mass of atoms initially contributes an increase of sensitivity of 10^{10}, but that the low atomic beam intensity, compared to photon beams, removes some of this advantage, as does the reduced number of round trips possible in an atom interferometer. Nevertheless, a typical factor of a 10^4 increase in rotation sensitivity can still be expected using atoms rather than photons.

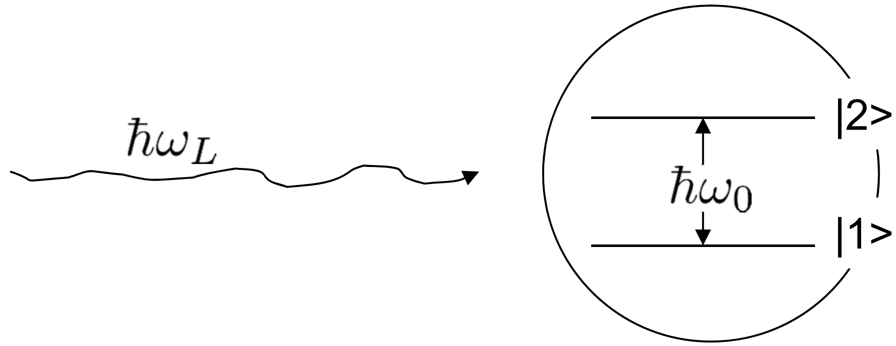
	Matter	Laser	Matter-to-light sensitivity factor
Mass factor	$\sim 10^4$ MeV	~ 1 eV	$\sim 10^{10}$
Flux	$\rho v A \sim 10^{10} \times 10^4 \times 10^{-2} = 10^{12} \frac{\text{particles}}{\text{sec}}$	$\frac{P}{\hbar\nu} \sim \frac{10^{-3}}{10^{-19}} = 10^{16} \frac{\text{photons}}{\text{sec}}$	$\sim 10^{-2}$
Round trips	~ 1	$\sim 10^4$	$\sim 10^{-4}$

Outline

- Interaction of two level atom with single mode field
    - Schrodinger Eq.
    - Density Matrix
- Rabi flopping
- Ramsey interference
- Spin echo
- Two time correlation functions



Hamiltonian



$$\mathcal{H} = \hbar\omega_0 |2><2| + \sum_{k,s} \hbar\omega_{k,s}\hat{n}_{k,s} - \hat{\mu}\cdot\hat{E}$$

Fully quantum mechanical

$$\mathcal{H} = \hbar\omega_0 |2><2| - \hat{\mu}\cdot E$$

Semi-classical

$$\hat{\mu} = \mu_{12}|1><2| + \mu_{12}^{*}|2><1|$$

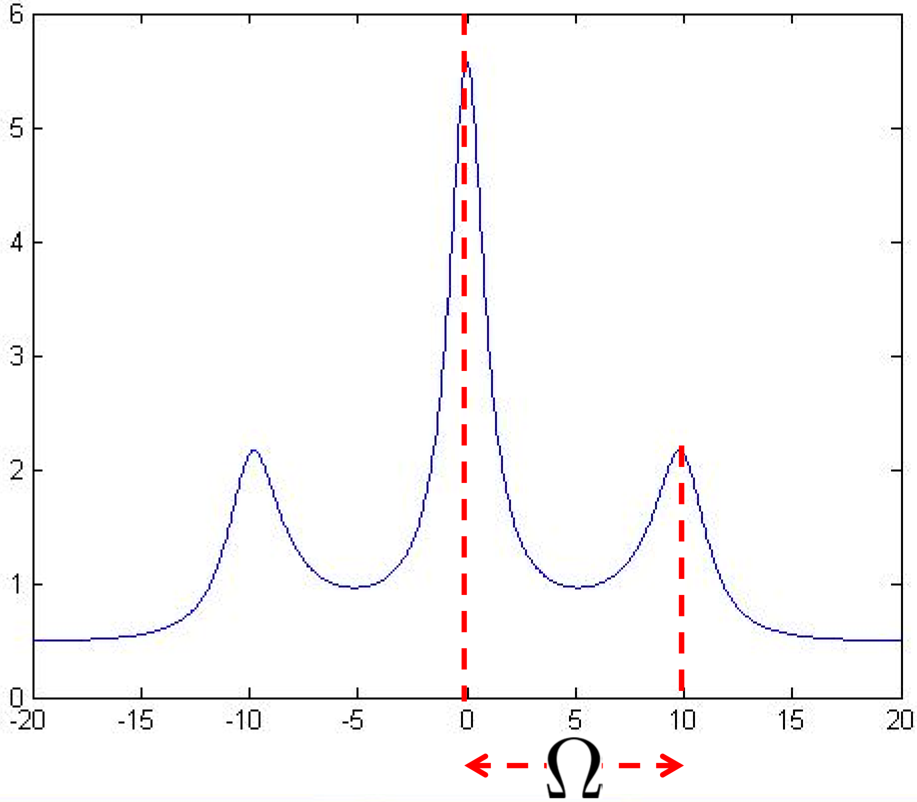
$$E = \mathcal{E}e^{-i\omega_L t} + c.c.$$

$$\begin{aligned} \mu\cdot E &= \mu_{12}\mathcal{E}^{*}e^{i\omega_L t}|1><2| + \mu_{12}^{*}\mathcal{E}e^{-i\omega_L t}|2><1| \\ &= \frac{1}{2}\hbar\Omega^{*}e^{i\omega_L t}|1><2| + \frac{1}{2}\hbar\Omega e^{-i\omega_L t}|2><1| \end{aligned}$$

Rabi frequency

$$\Omega = \frac{2\mu_{12}\mathcal{E}}{\hbar}$$

- As we'll see, this is the rate the atom oscillates between ground and excited states



The Schrodinger Equation

$$i\hbar \frac{\partial}{\partial t}\psi = H\psi$$

$$|\psi(t) > \quad = \quad c_1(t)|1 > + c_2(t)|2 >$$

$$\mathcal{H}|\psi(t) >= \left[\hbar\omega_o|2\rangle\langle 2| - \frac{\imath}{2}\hbar\Omega^* e^{\imath\omega_L t}|1\rangle\langle 2| - \frac{\imath}{2}\hbar\Omega e^{-\imath\omega_L t}|2\rangle\langle 1\right]$$
$$\times \left[c_1(t)|1\rangle + c_2(t)|2\rangle\right]$$
$$= \quad \hbar\omega_0 c_2(t)|2 > -\frac{i}{2}\hbar\Omega^* e^{i\omega_L t}c_2(t)|1 > -\frac{1}{2}\hbar\Omega e^{-i\omega_L t}c_1(t)|1 >$$

$$i\hbar \frac{\partial}{\partial t}|\psi(t) > \quad = \quad i\hbar\dot{c}_1(t)|1 > + i\hbar\dot{c}_2(t)|2 >$$

Some more algebra

$$
\begin{aligned}
i\hbar\dot{c}_1(t)|1> + i\hbar\dot{c}_2(t)|2> &= \hbar\omega_0 c_2(t)|2> - \frac{i}{2}\hbar\Omega^* e^{i\omega_L t} c_2(t)|1> \\
&\quad - \frac{1}{2}\hbar\Omega e^{-i\omega_L t} c_1(t)|2>
\end{aligned}
$$

$$
\begin{aligned}
\dot{c}_1(t) &= \frac{i}{2}\Omega^* e^{i\omega_L t} c_2(t) \\
\dot{c}_2(t) &= -i\omega_0 c_2(t) + \frac{i}{2}\Omega e^{-i\omega_L t} c_1(t)
\end{aligned}
$$

In principle, numerically integrable

Change to rotating frame

Some change into frame rotating at a rate ω_o

We will change into frame rotating at a rate ω_L

$$\begin{aligned} c_1(t) &= \tilde{c}_1(t) \\ c_2(t) &= \tilde{c}_2(t)e^{-\imath\omega_L t} \end{aligned}$$

Final set of equations

NAVAIR

$$\begin{cases} i\dot{\tilde{c}}_1(t) & = & -\frac{1}{2}\Omega^*\tilde{c}_2(t) \\ i\dot{\tilde{c}}_2(t) & = & (\omega_0 - \omega_L)\tilde{c}_2(t) - \frac{1}{2}\Omega\tilde{c}_1(t) \end{cases}$$

$$\delta = \omega_L - \omega_o$$

Laser detuning from atomic resonance

$$\begin{cases} \dot{\tilde{c}}_1(t) & = & 0\tilde{c}_1(t) + \frac{i}{2}\Omega^*\tilde{c}_2(t) \\ \dot{\tilde{c}}_2(t) & = & \frac{i}{2}\Omega\tilde{c}_1(t) + i\delta\tilde{c}_2(t) \end{cases}$$

$$\dot{\psi}(t) = L\psi(t)$$

where $L = \begin{pmatrix} 0 & +\frac{i}{2}\Omega^* \\ \frac{i}{2}\Omega & i\delta \end{pmatrix}$ $\quad \psi(t) = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$

Solving the equations

Solving this system by eigenvalue method

$$\left| \begin{pmatrix} -\lambda & +\frac{i}{2}\Omega^* \\ \frac{i}{2}\Omega & i\delta - \lambda \end{pmatrix} \right| = 0 \qquad \Omega' = \sqrt{\Omega^2 + \delta^2}$$

Generalized Rabi frequency

$$\lambda = \frac{i\delta \pm \sqrt{(-i\delta)^2 - (4)(1)(|\Omega|^2)}}{2} = \frac{i\delta \pm i\sqrt{|\Omega|^2 + \delta^2}}{2}$$

$$\lambda_+, \quad \underbrace{\begin{pmatrix} \Omega^*/(\Omega' + \delta) \\ 1 \end{pmatrix}}_{\psi_+}$$

$$\lambda_-, \quad \underbrace{\begin{pmatrix} -\Omega^*/(\Omega' - \delta) \\ 1 \end{pmatrix}}_{\psi_-}$$

General solution

$$\psi(t) = Ae^{\lambda_+ t}\psi_+ + Be^{\lambda_- t}\psi_-$$

$$\begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix} = Ae^{i(\Omega'+\delta)t/2} \begin{pmatrix} \Omega^+/(\Omega'+\delta) \\ 1 \end{pmatrix} + Be^{-i(\Omega'-\delta)t/2} \begin{pmatrix} -\Omega^*/(\Omega'-\delta) \\ 1 \end{pmatrix}$$

$$B = \left(\frac{\Omega'-\delta}{2\Omega'}\right) c_2(0) - \frac{\Omega}{2\Omega'} c_1(0)$$

$$A = \frac{\Omega'+\delta}{2\Omega'} c_2(0) + \frac{\Omega}{2\Omega'} c_1(0)$$

Most general form

$$c_1(t) = \left[\frac{|\Omega|^2}{2\Omega'(\Omega'+\delta)}c_1(0) + \frac{\Omega^*}{2\Omega'}c_2(0)\right]e^{i(\Omega'+\delta)t/2} + \left[\frac{|\Omega|^2}{2\Omega'(\Omega'-\delta)}c_1(0) - \frac{\Omega^*}{2\Omega'}c_2(0)\right]e^{-i(\Omega'\delta)t/2}$$

$$c_2(t) = \left[\frac{\Omega}{2\Omega'}c_1(0) + \frac{\Omega'+\delta}{2\Omega'}c_2(0)\right]e^{i(\Omega'+\delta)t/2} - \left[\frac{\Omega}{2\Omega'}c_1(0) - \frac{\Omega'-\delta}{2\Omega'}c_2(0)\right]e^{-i(\Omega'\delta)t/2}$$

Compact notation

- Amplitudes as if the atom was initially in the ground or excited state

Amplitude of ground state as if atom started in the ground state

$$\tilde{c}_1^g(t) = \frac{|\Omega|^2}{2\Omega'(\Omega'+\delta)} e^{\imath(\Omega'+\delta)t/2} + \frac{|\Omega|^2}{2\Omega'(\Omega'-\delta)} e^{-\imath(\Omega'-\delta)t/2},$$

$$\tilde{c}_2^g(t) = \frac{\Omega}{2\Omega'} e^{\imath(\Omega'+\delta)t/2} - \frac{\Omega}{2\Omega'} e^{\imath(\Omega'-\delta)t/2},$$

Compact notation

- Amplitudes as if the atom was initially in the ground or excited state

$$\tilde{c}_1^g(t) = \frac{|\Omega|^2}{2\Omega'(\Omega'+\delta)} e^{\imath(\Omega'+\delta)t/2} + \frac{|\Omega|^2}{2\Omega'(\Omega'-\delta)} e^{-\imath(\Omega'-\delta)t/2},$$

$$\tilde{c}_2^g(t) = \frac{\Omega}{2\Omega'} e^{\imath(\Omega'+\delta)t/2} - \frac{\Omega}{2\Omega'} e^{\imath(\Omega'-\delta)t/2},$$

Amplitude of excited state as if atom started in the ground state

Compact notation

- Amplitudes as if the atom was initially in the ground or excited state

$$\tilde{c}_1^g(t) = \frac{|\Omega|^2}{2\Omega'(\Omega'+\delta)} e^{\imath(\Omega'+\delta)t/2} + \frac{|\Omega|^2}{2\Omega'(\Omega'-\delta)} e^{-\imath(\Omega'-\delta)t/2},$$

$$\tilde{c}_2^g(t) = \frac{\Omega}{2\Omega'} e^{\imath(\Omega'+\delta)t/2} - \frac{\Omega}{2\Omega'} e^{\imath(\Omega'-\delta)t/2},$$

- Similarly

$$\tilde{c}_1^e(t) = \frac{\Omega^*}{2\Omega'} e^{\imath(\Omega'+\delta)t/2} - \frac{\Omega^*}{2\Omega'} e^{-\imath(\Omega'+\delta)t/2},$$

$$\tilde{c}_2^e(t) = \frac{\Omega'+\delta}{2\Omega'} e^{\imath(\Omega'+\delta)t/2} + \frac{\Omega'-\delta}{2\Omega'} e^{-\imath(\Omega'+\delta)t/2}$$

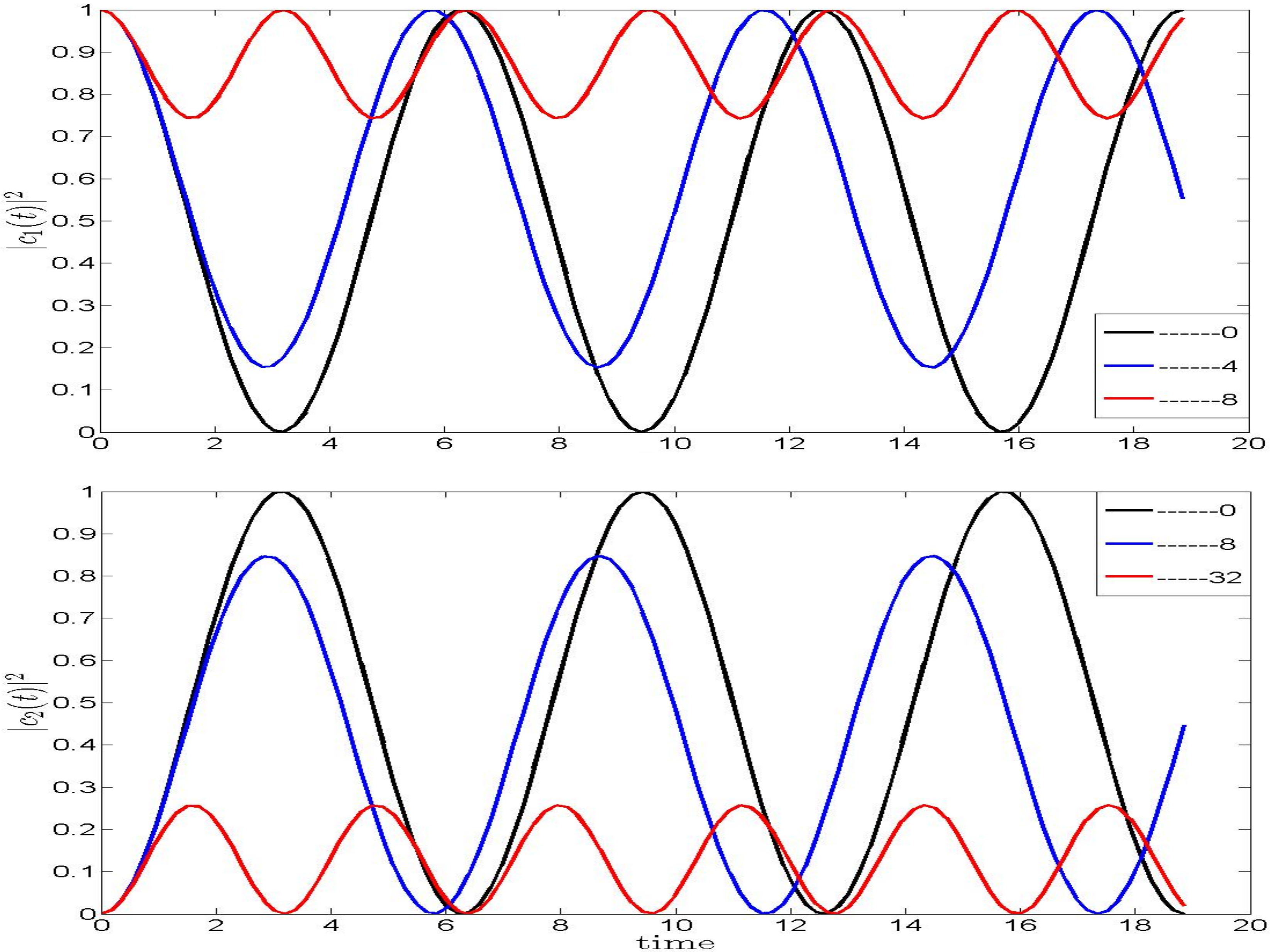
Finally….

- Basically, we now have the probability amplitudes to find the atom in the excited or ground states as a function of time

$$
\begin{aligned}
\tilde{c}_1(t) &= \tilde{c}_1^g(t)\ \tilde{c}_1(0) + \tilde{c}_1^e(t)\ \tilde{c}_2(0), \\
\tilde{c}_2(t) &= \tilde{c}_2^g(t)\ \tilde{c}_1(0) + \tilde{c}_2^e(t)\ \tilde{c}_2(0).
\end{aligned}
$$

Now…..on to some physics

What does this look like?



Simple solution

- For atom initially in the ground state

$$\tilde{c}_2^g(t) = \frac{\Omega}{2\Omega'} e^{\imath(\Omega'+\delta)t/2} - \frac{\Omega}{2\Omega'} e^{-\imath(\Omega'-\delta)t/2}$$

$$= \frac{\imath\Omega}{\Omega'} \sin(\Omega' t/2) e^{\imath\delta t/2}$$

- Therefore

$$|\tilde{c}_2^g(t)|^2 = \left|\frac{\Omega}{\Omega'}\right|^2 \sin^2(\Omega' t/2)$$

$$= \frac{1}{2}\left|\frac{\Omega}{\Omega'}\right|^2 (1 - \cos\Omega' t)$$

Oscillates at exactly Ω'

Steady state…

- As constructed, there is no “steady state”
  - System continues to oscillate forever

We have ignored spontaneous emission!

Inclusion of spontaneous emission

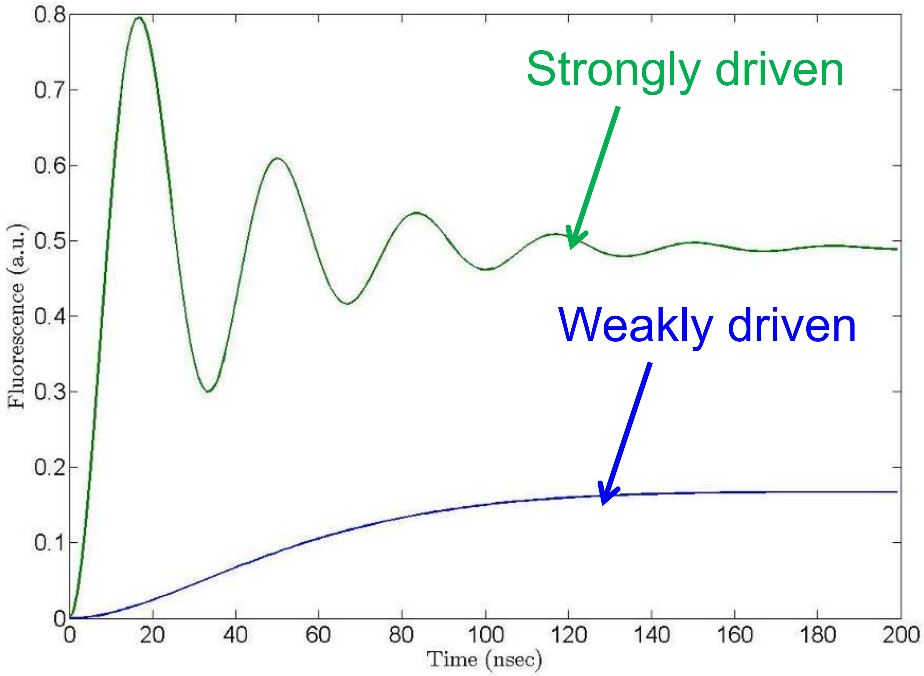
- Put in "by hand"
  - We'll take the excited state decay rate to be 2β
  - We'll take the coherence decay rate to be β

- Really requires a density matrix approach which we will not cover in this lecture.

But here we'll skip to the end…

Dynamics

- No "nice" analytic solution exists
- Analytic solutions exist if…
  - Spontaneous emission is ignored (as shown before)
  - Detuning is taken to be zero



Steady State

$$|c_2(t)|^2 = \frac{\frac{1}{4}\frac{\Omega^2}{\beta^2}}{\frac{1}{2}\frac{\Omega^2}{\beta^2} + 1 + \frac{\delta^2}{\beta^2}} \to \frac{1}{2}\ as\ \Omega \to \infty$$

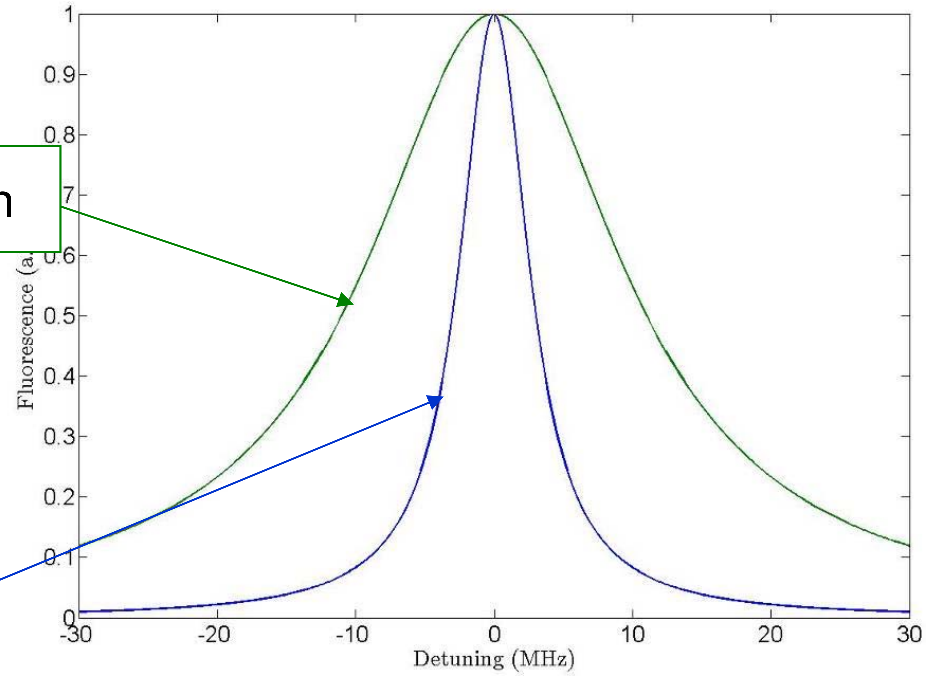
Re-arranging

$$|c_2(t)|^2 = \frac{\frac{1}{4}\Omega^2}{\left(\beta^2 + \frac{1}{2}\Omega^2\right) + \delta^2}$$

Powerbroadened Linewidth

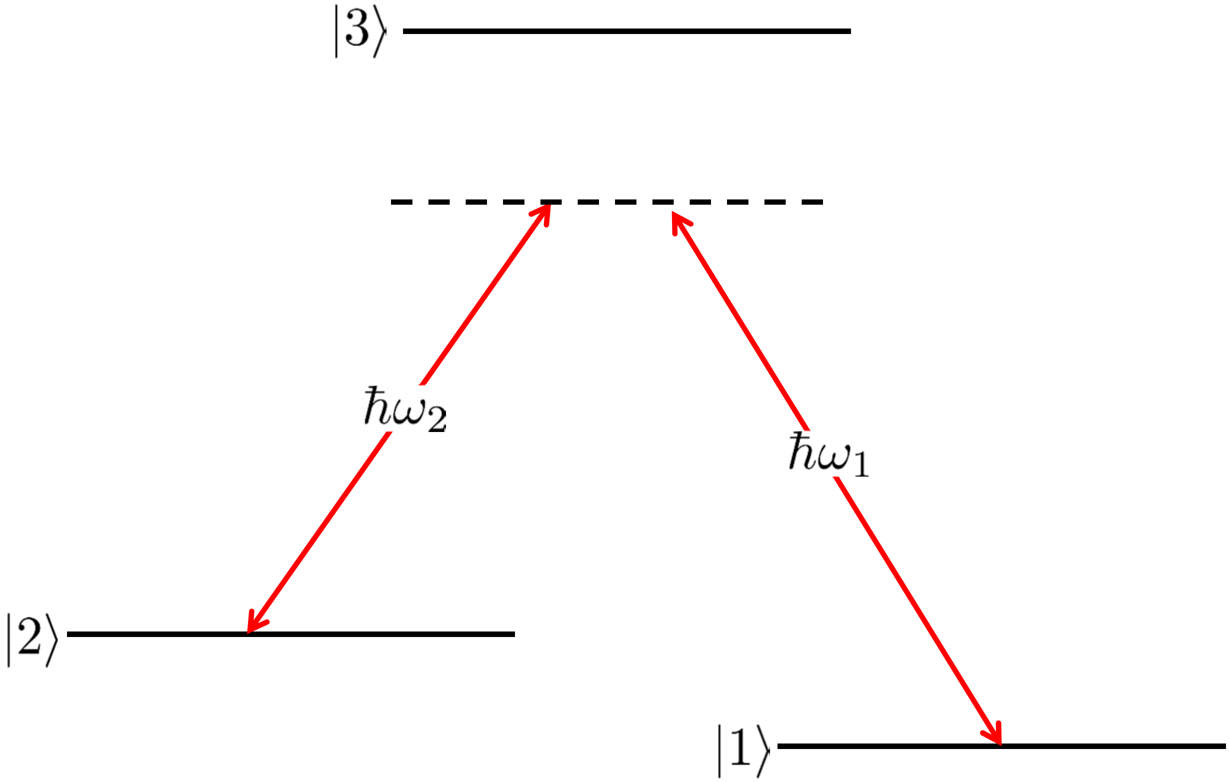
$$\beta_{effective} = \beta\sqrt{1 + \frac{1}{2}\frac{\Omega^2}{\beta}}$$

Natural Linewidth



Justification for ignoring spontaneous emission

Three level system



$$\hat{\mathcal{H}} = \hbar\Delta|2\rangle\langle 2| + \hbar\omega_{13}|3\rangle\langle 3| - \hat{\mu}\cdot E$$

$$\begin{aligned}\hat{\mu}\cdot E \quad &= \quad \mu_{13}\mathcal{E}_1^* e^{\imath\omega_1 t}|1\rangle\langle 3| + \mu_{23}\mathcal{E}_2^* e^{\imath\omega_2 t}|2\rangle\langle 3| \\ &\quad\quad + \mu_{13}^*\mathcal{E}_1 e^{-\imath\omega_1 t}|3\rangle\langle 1| + \mu_{23}^*\mathcal{E}_2 e^{-\imath\omega_2 t}|3\rangle\langle 2|\end{aligned}$$

System of equations

$$|\psi(t) = c_1(t)|1\rangle + c_2(t)|2\rangle + c_3(t)|3\rangle$$

$$\vdots$$

$$\dot{\tilde{c}}_1(t) = \frac{\imath}{2}\Omega_1^*\tilde{c}_3(t)$$

$$\dot{\tilde{c}}_2(t) = -\imath\left(\delta_2 - \delta_1\right) + \frac{\imath}{2}\Omega_2^*\tilde{c}_3(t)$$

$$\dot{\tilde{c}}_3(t) = \imath\delta_1\tilde{c}_3(t) + \frac{\imath}{2}\Omega_1\tilde{c}_1(t) + \frac{\imath}{2}\Omega_2\tilde{c}_2(t)$$

Adiabatically eliminate the 3rd level

Take $\dot{\tilde{c}}_3(t) = 0$ and solve for $\tilde{c}_3(t)$

$$\tilde{c}_3(t) = -\frac{1}{2}\frac{\Omega_1}{\delta_1}\tilde{c}_1(t) - \frac{1}{2}\frac{\Omega_2}{\delta_1}\tilde{c}_2(t)$$

Substitute back into equations for $c_1(t)$ and $c_2(t)$

$$\dot{\tilde{c}}_1(t) = -\frac{\imath}{4}\frac{|\Omega_1|^2}{\delta_1}\tilde{c}_1(t) - \imath\frac{\Omega_1^*\Omega_2}{\delta_1}\tilde{c}_2(t)$$

$$\dot{\tilde{c}}_2(t) = \imath\left(\delta_2 - \delta_1\right)\tilde{c}_2(t) - \frac{\imath}{4}\frac{\Omega_1\Omega_2^*}{\delta_1}\tilde{c}_1(t) - \frac{\imath}{4}\frac{|\Omega_2|^2}{\delta_2}\tilde{c}_2(t)$$

These are close to the final solution,

Let's do one more transformation to drive the point home

One more transformation

$$\tilde{c}_1(t) = \tilde{c}_1(t) e^{-\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1} t} \qquad \tilde{c}_2(t) = \tilde{d}_2(t) e^{-\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1} t}$$

Adiabatically eliminated three level equations

$$\dot{\tilde{d}}_1(t) = -\imath \frac{\Omega_1^* \Omega_2}{4\delta_1} \tilde{d}_2 \qquad \dot{\tilde{d}}_2(t) = \imath \left[(\delta_1 - \delta_2) + \left(\frac{|\Omega_1|^2}{4\delta_1} - \frac{|\Omega_2|^2}{4\delta_2} \right) \right] \tilde{d}_2 - \frac{\imath}{4} \frac{\Omega_1 \Omega_2^*}{\delta_1} \tilde{d}_1$$

Original two level equations

$$\dot{\tilde{c}}_1(t) = \frac{\imath}{2} \Omega^* \tilde{c}_2(t) \qquad \dot{\tilde{c}}_2(t) = \imath \delta \tilde{c}_2(t) + \frac{\imath}{2} \Omega \tilde{c}_1(t)$$

Functionally, the same form!

One more transformation

$$\tilde{d}_1(t) = \tilde{c}_1(t) e^{\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1}t} \qquad \tilde{d}_2(t) = \tilde{c}_2(t) e^{\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1}t}$$

Adiabatically eliminated three level equations

$$\dot{\tilde{d}}_1(t) = -\imath\frac{\Omega_1^*\Omega_2}{4\delta_1}\tilde{d}_2 \qquad \dot{\tilde{d}}_2(t) = \imath\left[(\delta_1 - \delta_2) + \left(\frac{|\Omega_1|^2}{4\delta_1} - \frac{|\Omega_2|^2}{4\delta_2}\right)\right]\tilde{d}_2 - \frac{\imath}{4}\frac{\Omega_1\Omega_2^*}{\delta_1}\tilde{d}_1$$

Original two level equations

$$\dot{\tilde{c}}_1(t) = \frac{\imath}{2}\Omega^*\tilde{c}_2(t) \qquad \dot{\tilde{c}}_2(t) = \imath\delta\tilde{c}_2(t) + \frac{\imath}{2}\Omega\tilde{c}_1(t)$$

$$\Omega = -\frac{\Omega_1\Omega_2^*}{2\delta_1} \rightarrow \text{Raman Rabi frequency}$$

One more transformation

$$\tilde{d}_1(t) = \tilde{c}_1(t)e^{\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1}t} \qquad \tilde{d}_2(t) = \tilde{c}_2(t)e^{\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1}t}$$

Adiabatically eliminated three level equations

$$\dot{\tilde{d}}_1(t) = -\imath\frac{\Omega_1^*\Omega_2}{4\delta_1}\tilde{d}_2 \qquad \dot{\tilde{d}}_2(t) = \imath\left[(\delta_1 - \delta_2) + \left(\frac{|\Omega_1|^2}{4\delta_1} - \frac{|\Omega_2|^2}{4\delta_2}\right)\right]\tilde{d}_2 - \frac{\imath}{4}\frac{\Omega_1\Omega_2^*}{\delta_1}\tilde{d}_1$$

Original two level equations

$$\dot{\tilde{c}}_1(t) = \frac{\imath}{2}\Omega^*\tilde{c}_2(t) \qquad \dot{\tilde{c}}_2(t) = \imath\delta\tilde{c}_2(t) + \frac{\imath}{2}\Omega\tilde{c}_1(t)$$

Two photon detuning $\delta_2 - \delta_1$ plays the role of detuning

One more transformation

$$\tilde{d}_1(t) = \tilde{c}_1(t) e^{\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1}t} \qquad \tilde{d}_2(t) = \tilde{c}_2(t) e^{\frac{\imath}{4}\frac{|\Omega|^2}{\delta_1}t}$$

Adiabatically eliminated three level equations

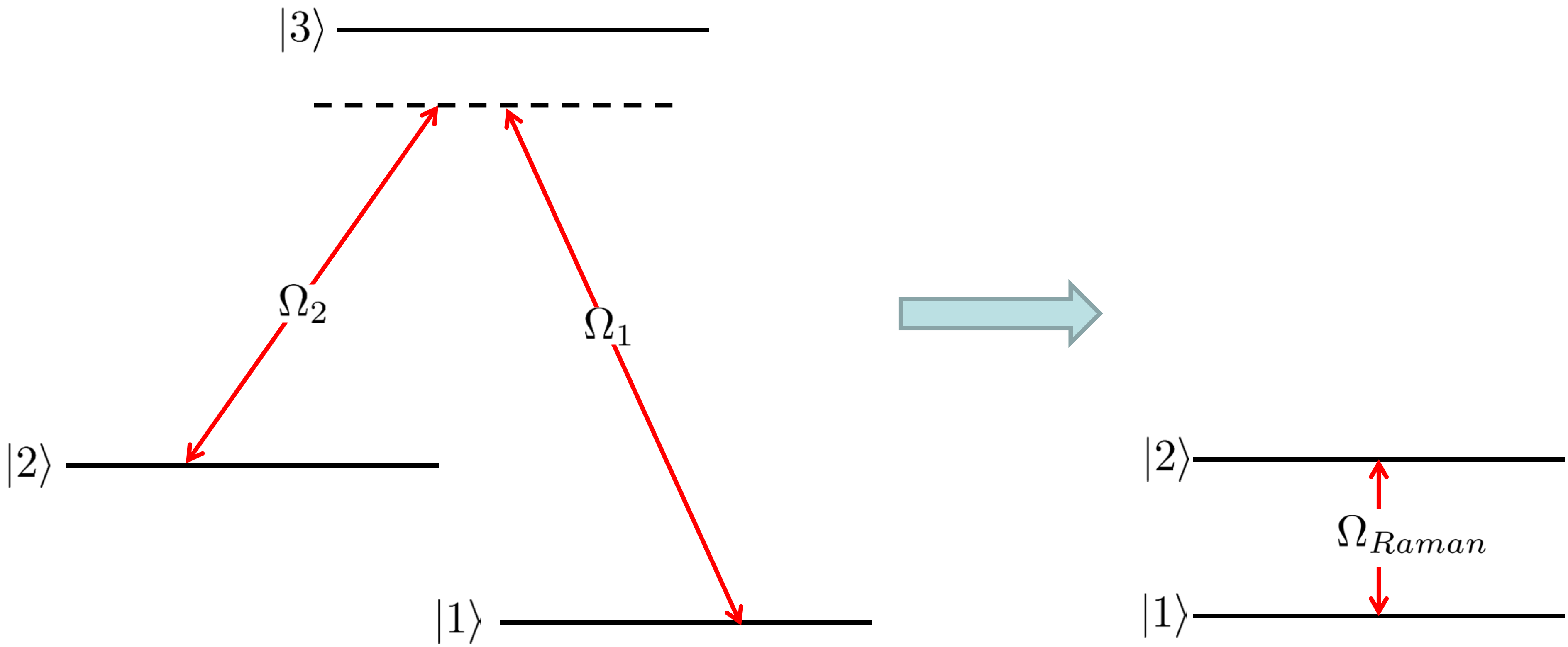
$$\dot{\tilde{d}}_1(t) = -\imath \frac{\Omega_1^* \Omega_2}{4\delta_1} \tilde{d}_2 \qquad \dot{\tilde{d}}_2(t) = \imath \left[(\delta_1 - \delta_2) + \left(\frac{|\Omega_1|^2}{4\delta_1} - \frac{|\Omega_2|^2}{4\delta_2} \right) \right] \tilde{d}_2 - \frac{\imath}{4} \frac{\Omega_1 \Omega_2^*}{\delta_1} \tilde{d}_1$$

Original two level equations

$$\dot{\tilde{c}}_1(t) = \frac{\imath}{2} \Omega^* \tilde{c}_2(t) \qquad \dot{\tilde{c}}_2(t) = \imath \delta \tilde{c}_2(t) + \frac{\imath}{2} \Omega \tilde{c}_1(t)$$

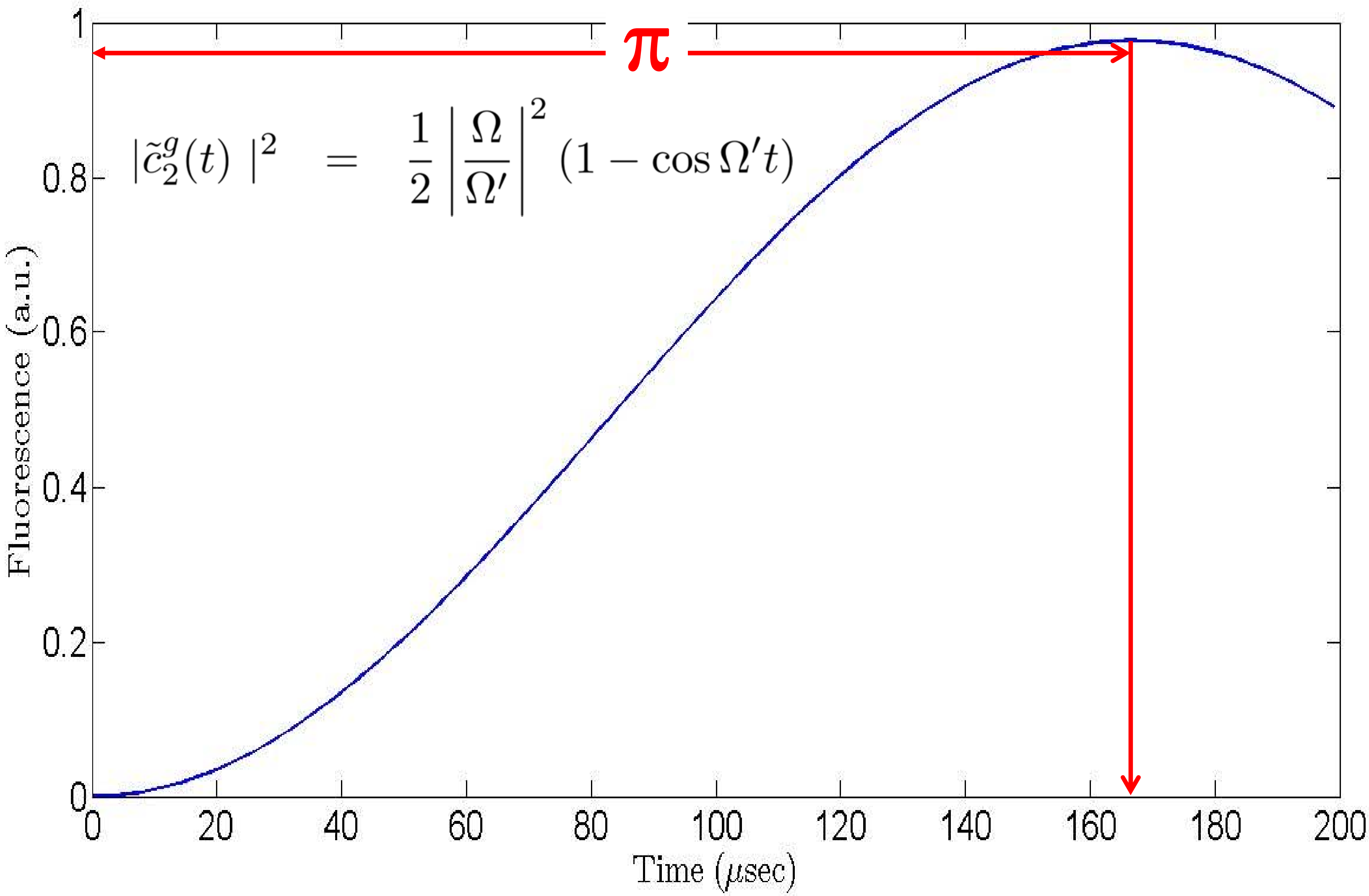
AC Stark shift

Going back…

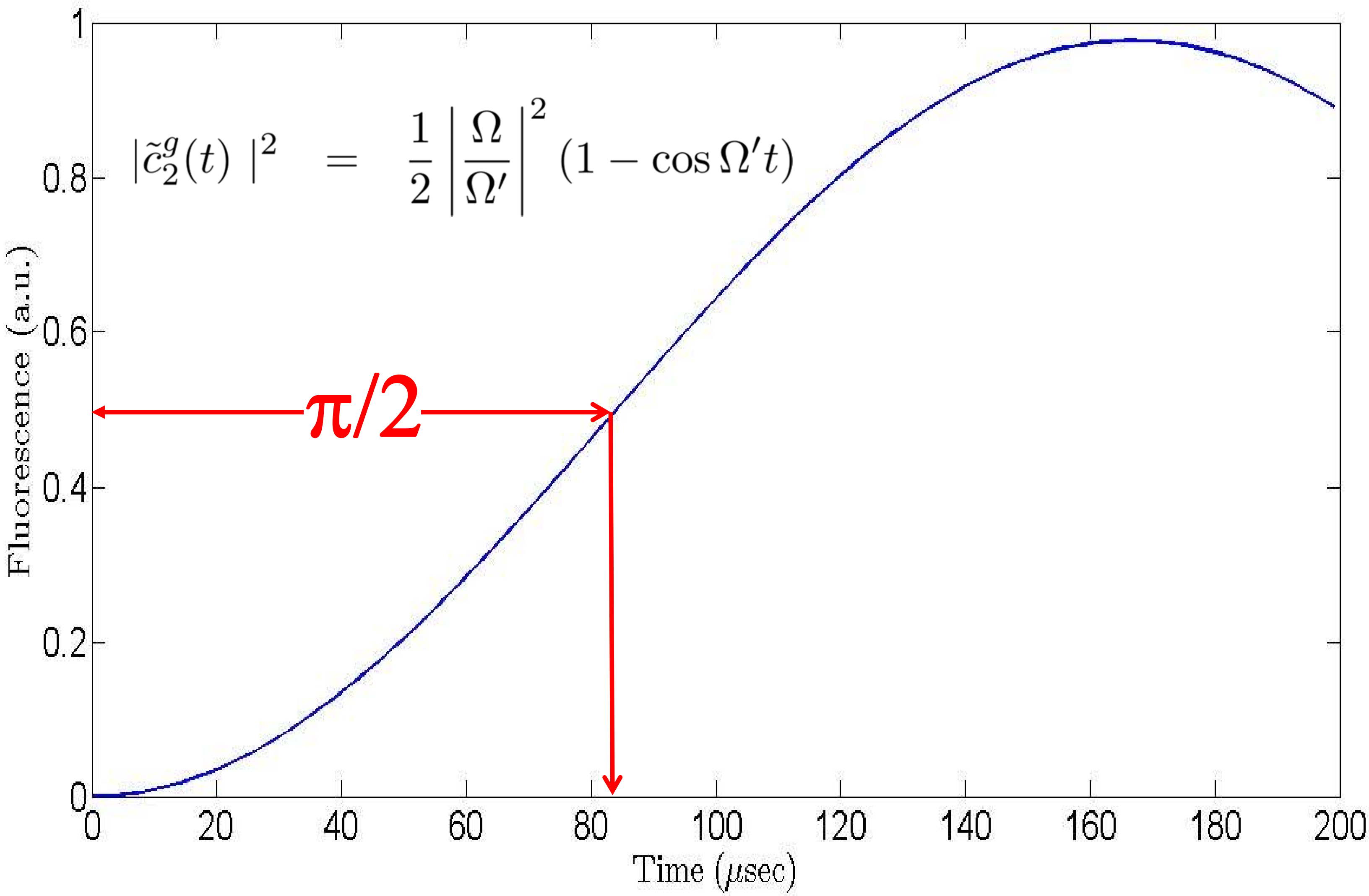


Now states $|1\rangle$ and $|2\rangle$ are ground states!
This justifies ignoring spontaneous emission

Definition of π pulse



Definition of π/2 pulse



Single pulse experiments

- Consider an atom initially in the ground state

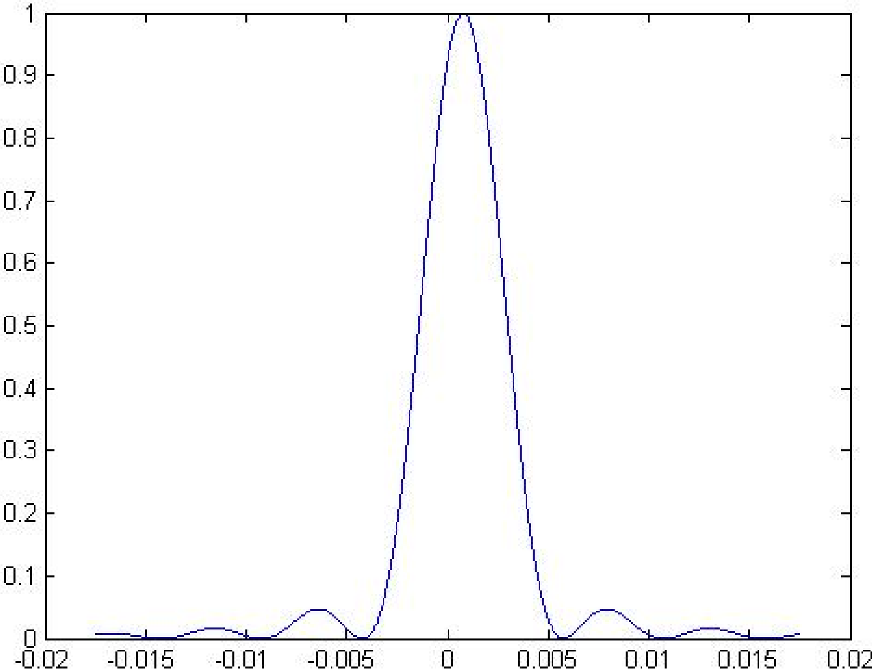
$$|\psi(0)\rangle = |1\rangle$$

- Apply a pulse that is "nominally" a $\pi/2$ pulse
  - Denote that time by $T_{\pi/2}$

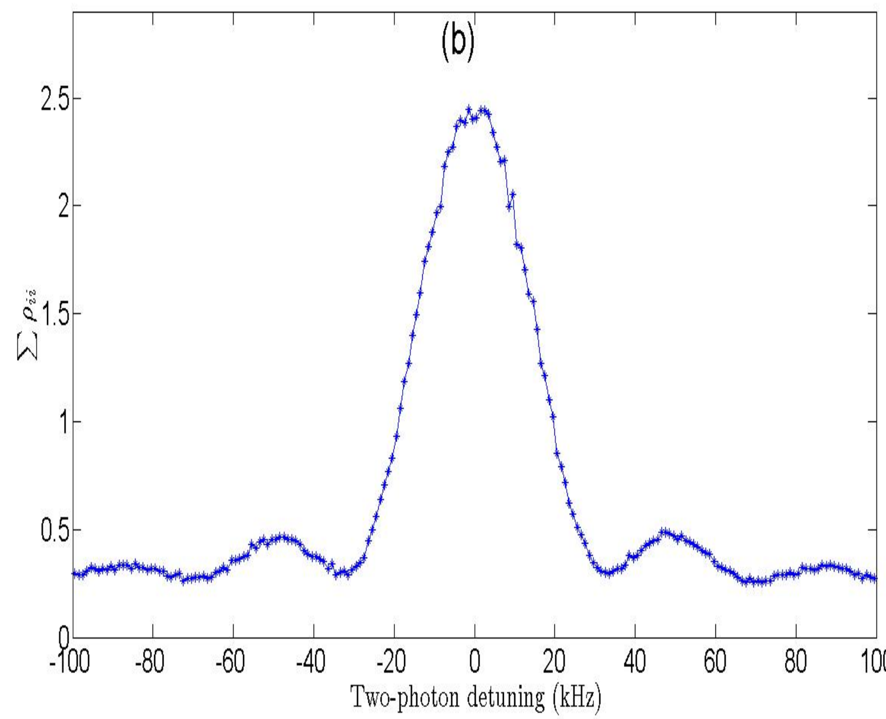
$$|\psi(t > T_{\pi/2})\rangle = \tilde{c}_1^g(T_{\pi/2})|1\rangle + \tilde{c}_2^g(T_{\pi/2})|2\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$$ (for perfect $\pi/2$ pulses)

$$P_2(t > T_{\pi/2}) = \left|\langle 2|\psi(t)\rangle\right|^2 = \left|\tilde{c}_2^g(T_{\pi/2})\right|^2$$

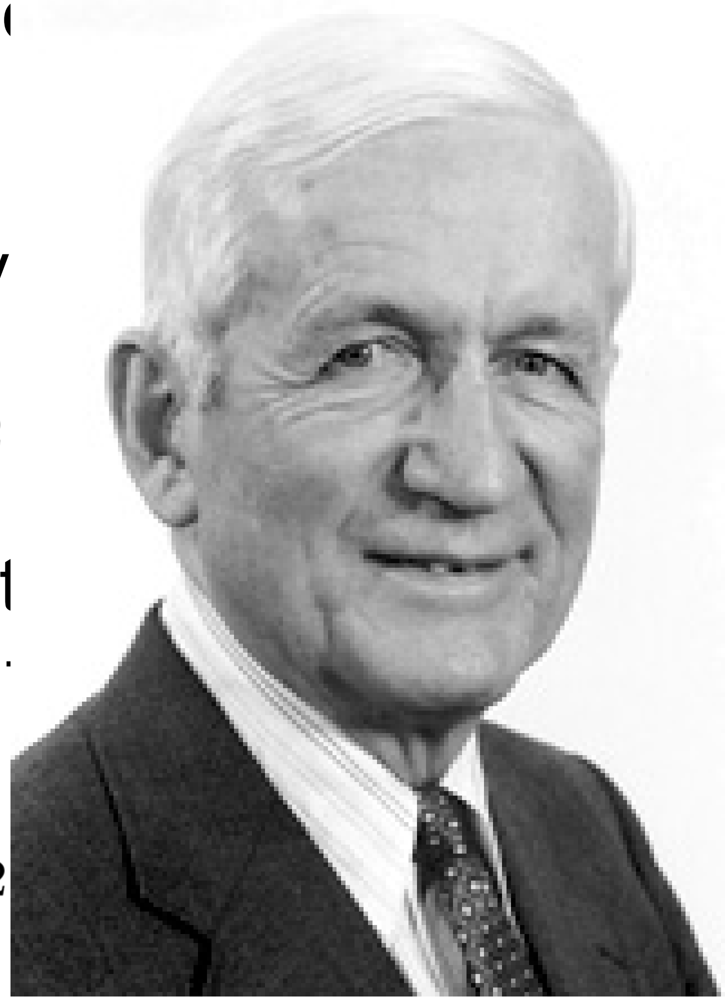


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
0.02



(b)
$\sum \rho_{ii}$
2.5
2
1.5
1
0.5
0
-100
-80
-60
-40
-20
0
20
40
60
80
100
Two-photon detuning (kHz)

Double pulse sequence-"Ramsey"



- As before, b he ground state

- Again, apply th $T_{\pi/2}$

$|\psi(t > T_{\pi/2}$ $T_{\pi/2})|2\rangle$

- Now, allow at r a time T_1
  (taking $|1\rangle$ energy)

$|\psi(t > T_{\pi/2}$ $\tilde{c}_2^g(T_{\pi/2})|2\rangle$

Double pulse sequence-“Ramsey”

- Now apply second pulse (assumed identical to the first one)

$$|\psi(T_{\pi/2} + T_1 + T_{\pi/2})\rangle =$$

$$\left[\tilde{c}_1^g(T_{\pi/2})\ \tilde{c}_1^g(T_{\pi/2})\ + e^{\imath\delta T_1}\tilde{c}_2^g(T_{\pi/2})\ \tilde{c}_1^e(T_{\pi/2})\ \right]|1\rangle$$

$$+\left[\tilde{c}_1^g(T_{\pi/2})\ \tilde{c}_2^g(T_{\pi/2})\ + e^{\imath\delta T_1}\tilde{c}_2^g(T_{\pi/2})\ \tilde{c}_2^e(T_{\pi/2})\ \right]|2\rangle.$$

$$P_2(t) = \left|\langle 2|\psi(T_{\pi/2} + T_1 + T_{\pi/2})\rangle\right|^2$$

$$= \left|\tilde{c}_1^g(T_{\pi/2})\ \tilde{c}_2^g(T_{\pi/2})\ + e^{\imath\delta T_1}\tilde{c}_2^g(T_{\pi/2})\ \tilde{c}_2^e(T_{\pi/2})\ \right|^2.$$

Simplification

NAVAIR

$$P_2(t) = \left|\tilde{c}_1^g(T_{\pi/2})\ \tilde{c}_2^g(T_{\pi/2})\ + e^{\imath\delta T_1}\tilde{c}_2^g(T_{\pi/2})\ \tilde{c}_2^e(T_{\pi/2})\ \right|^2 .$$

$$= \left|\tilde{c}_2^g(T_{\pi/2})\ \right|^2 \left|\tilde{c}_1^g(T_{\pi/2})\ + e^{\imath\delta T_1}\tilde{c}_2^e(T_{\pi/2})\ \right|^2$$

- By physics

$$\tilde{c}_2^e(T_{\pi/2})\ = \left(\tilde{c}_1^g(T_{\pi/2})\ \right)^*$$

$$P_2(t) = \left|\tilde{c}_2^g(T_{\pi/2})\ \right|^2 \left|\tilde{c}_1^g(T_{\pi/2})\ \right|^2 \left|1 + e^{\imath\phi}e^{\imath\delta T_1}\right|^2$$

Same function as before

Simplification

$$P_2(t) = \left| \tilde{c}_1^g(T_{\pi/2})\ \tilde{c}_2^g(T_{\pi/2})\ + e^{\imath\delta T_1} \tilde{c}_2^g(T_{\pi/2})\ \tilde{c}_2^e(T_{\pi/2})\ \right|^2 .$$

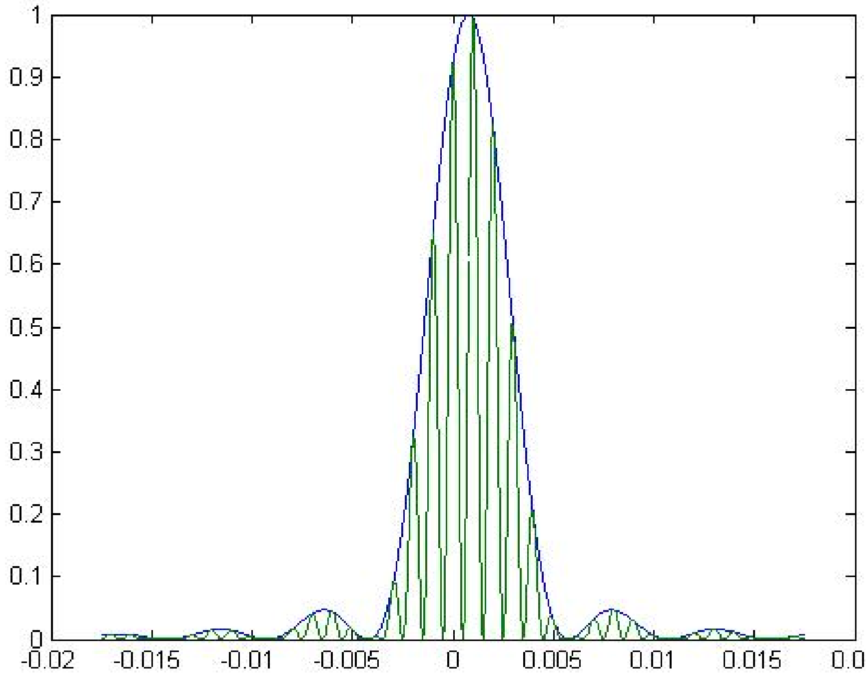
$$= \left| \tilde{c}_2^g(T_{\pi/2})\ \right|^2 \left| \tilde{c}_1^g(T_{\pi/2})\ + e^{\imath\delta T_1} \tilde{c}_2^e(T_{\pi/2})\ \right|^2$$

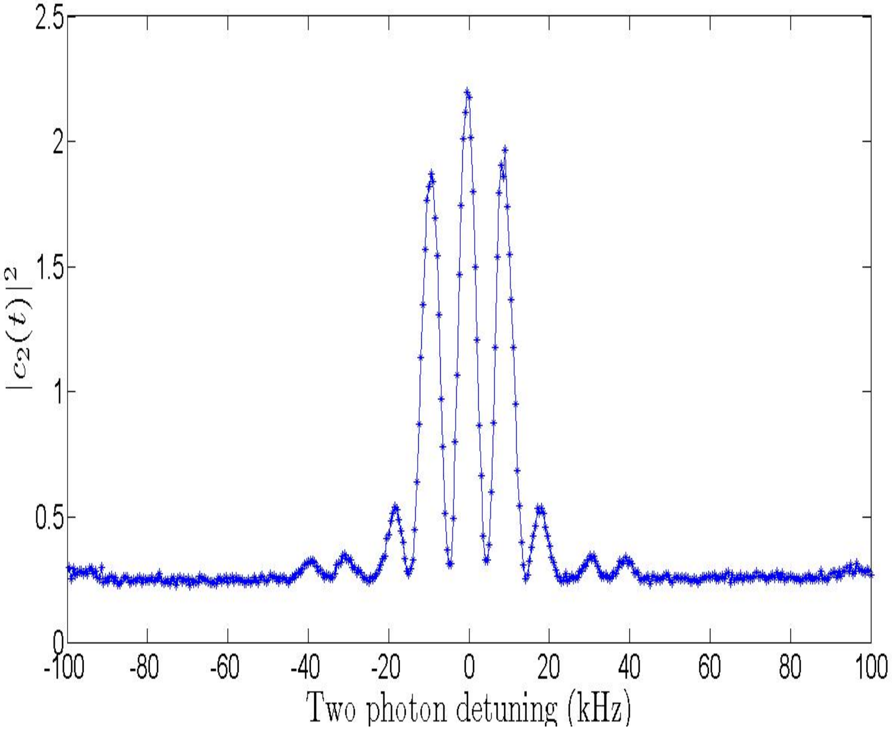
- By physics

$$\tilde{c}_2^e(T_{\pi/2})\ = \left(\tilde{c}_1^g(T_{\pi/2})\ \right)^*$$

$$P_2(t) = \left| \tilde{c}_2^g(T_{\pi/2})\ \right|^2 \left| \tilde{c}_1^g(T_{\pi/2})\ \right|^2 \left| 1 + e^{\imath\phi} e^{\imath\delta T_1} \right|^2$$

Interference!!!


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015


2.5
2
1.5
1
0.5
0
$|c_2(t)|^2$
-100
-80
-60
-40
-20
0
20
40
60
80
100
Two photon detuning (kHz)

Recall

Assume atom initially in the ground state $\psi(0) = |1\rangle$

After $\pi/2$ pulse: $|\psi(t) = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$

A "beam splitter"

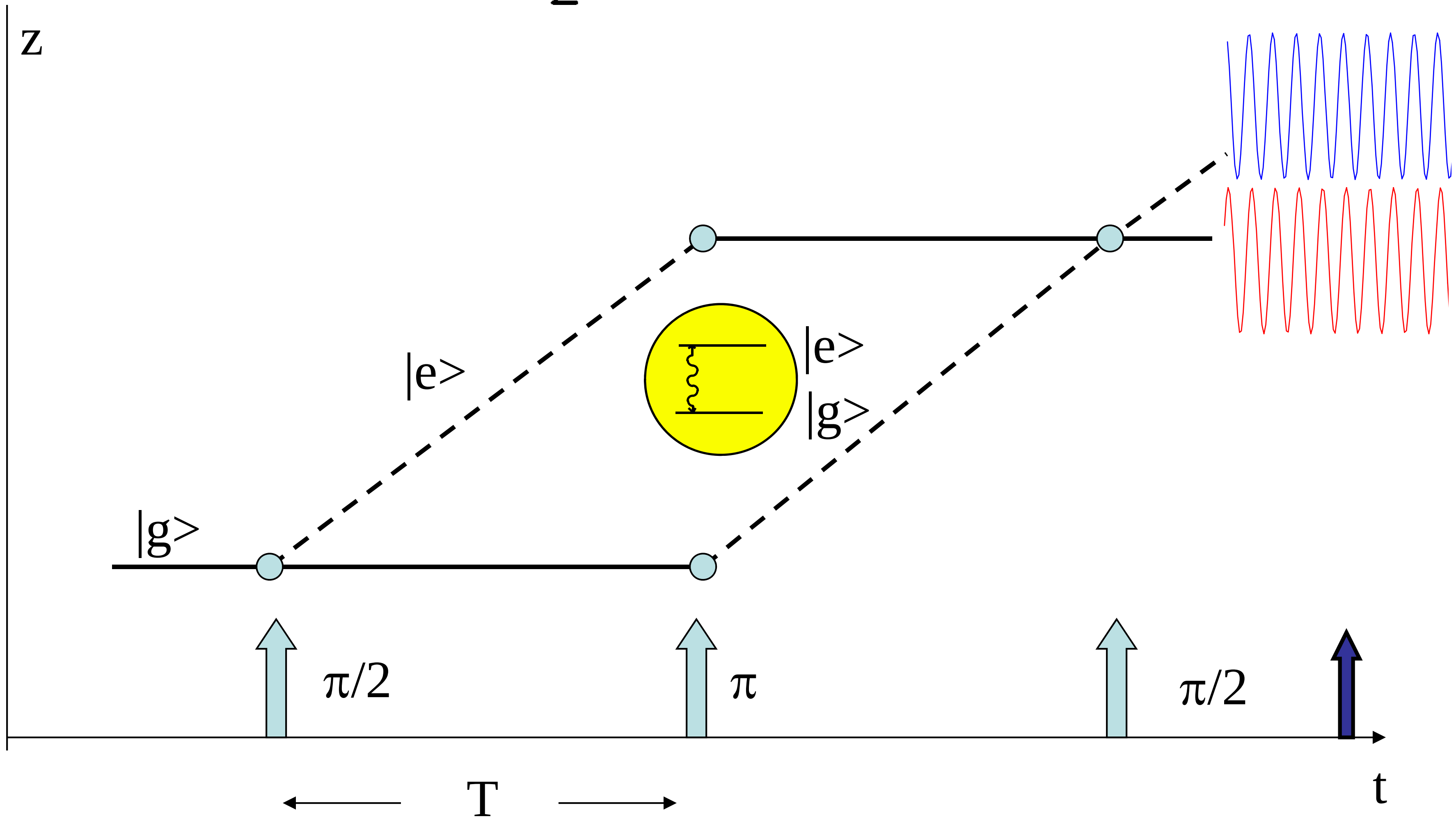
After a π pulse: $|1\rangle \rightarrow |2\rangle$

$|2\rangle \rightarrow |1\rangle$

A "mirror"

Overview of AI Sensors

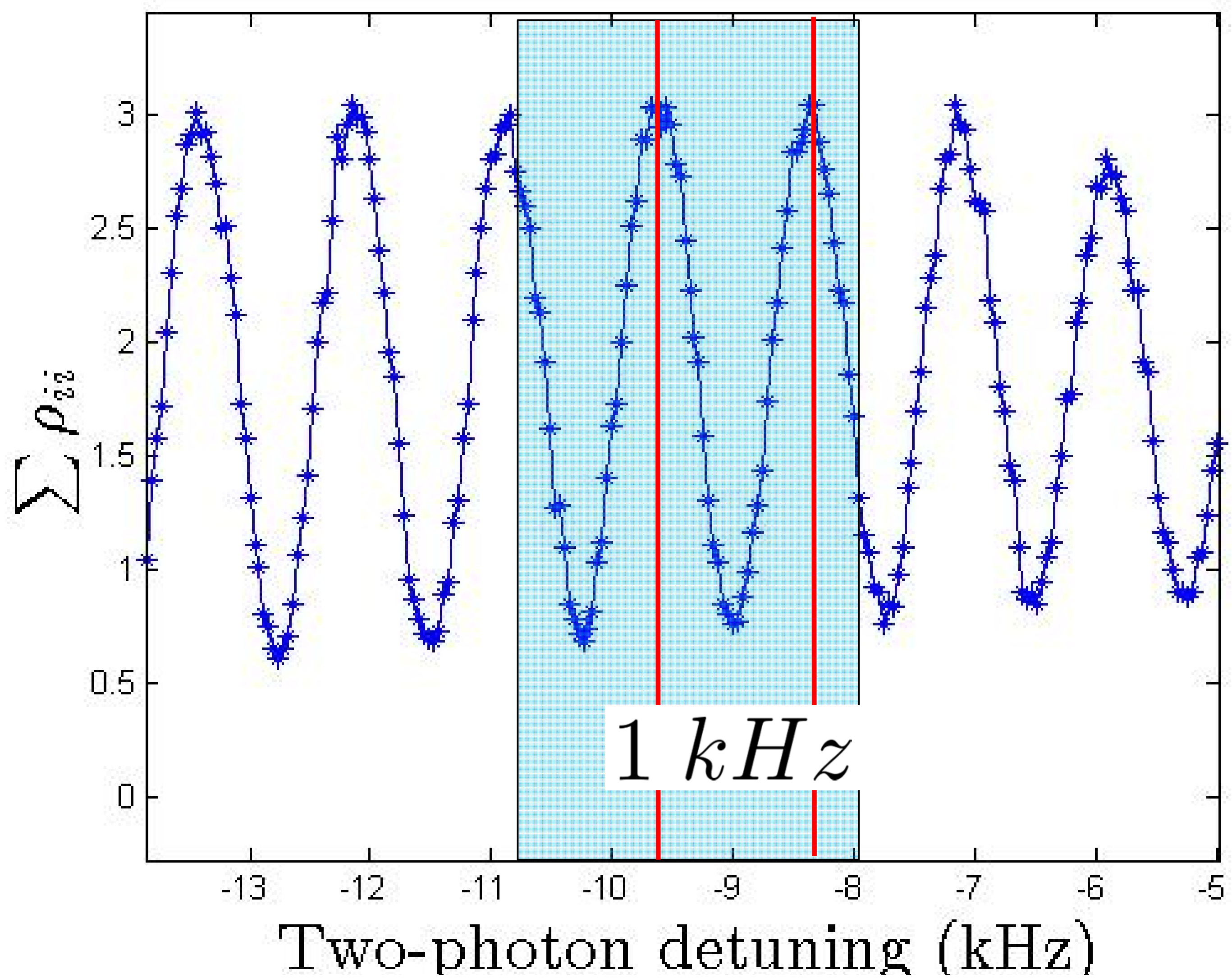
$$|c_{e,p+\hbar k}(2T+\tau)|^2 = \frac{1}{2}[1-\cos(\Delta\phi - \delta\tau/2)]$$



…and just for fun

Make T1 as long as possible

Even more fun….

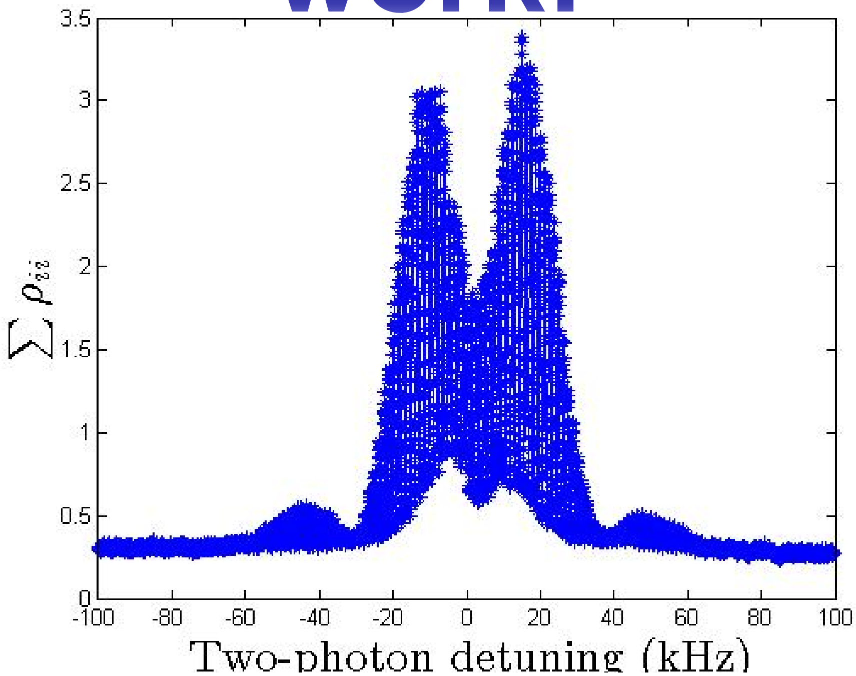


$$\text{Resonance width } \Delta\nu \sim 10^3 \text{ Hz}$$

$$\text{Optical frequencies } \nu \sim 10^{15} \text{ Hz}$$

$$\text{Precision of } \frac{\Delta\nu}{\nu} \sim 10^{-12}$$

Good enough for government work!



Questions?